# Polynomial

Support for Polynomial objects in JavaScript. When used with the Complex JavaScript Library the Complex JavaScript library needs to be loaded first.

#### Constructor

```
new Polynomial(coefficients...) // Invoked as a Constructor Polynomial(coefficients...) // Invoked as a Conversion
```

# Arguments

coefficients

Optional Polynomial Coefficients. *Coefficients* can be either JavaScript number, JavaScript Complex Object number, an array of numbers or another Polynomial objects.

#### Returns

Returns a normalized Polynomial object with an Array holding the coefficients. The coefficients in the Array can be normal JavaScript numbers or complex JavaScript numbers.

If *Polynomial* is invoked as a conversion the *coefficients* parameter is converted to a *Polynomial object* and returned.

If *coefficients* is undefined, an empty *Polynomial object* is returned.

Regardless if invoked as a new constructor or as a Conversion Polynomial constructor always return a Polynomial normalized object.

# Example:

### **Normalized Polynomial**

The Polynomial is normalized by eliminating leading zero coefficients and converting undefined coefficients to 0.

Notice that all properties or methods can work on both regular real coefficients or complex number coefficients or a mix of real and complex number coefficients. E.g.

# **Properties**

Properties	Description
array()	Return a Polynomial object as an array, where each array element is the coefficients of the polynomial in
	descending order of power
degree()	Returns the degree of the Polynomial
$getcoeff(x^{th})$	Return the coefficients belonging to x <sup>th</sup> power of the Polynomial
isComplex()	Return true if at least one coefficient in the Polynomial is a complex number
isReal()	Return true if all the coefficients in the Polynomial are only JavaScript numbers.
join(seperator)	Concatenate polynomial coefficients to form a string that is returned. If a <i>separator</i> is specified, each element is separated by the <i>separator</i>
monic()	Bring the polynomial into a monic form in which the leading coefficients is 1 by dividing the leading coefficient with all the other coefficients
normalize()	The Polynomial is normalized by eliminating trailing zero coefficients and converting undefined coefficients to 0. The normalized polynomial is returned
scale()	Scale the polynomial by multiplying all the coefficients with a factor. The factor can be automatic calculated.
$setcoeff(x^{th}, newcoeff)$	Set the coefficient belonging to the x <sup>th</sup> power of the polynomial to the <i>newcoeff</i> value
shift(no)	Do a Polynomial Taylor shift of no
simplify()	The Polynomial is simplified by reducing complex numbers with an imaginary part of zero to a real number
toExponential(digits)	Return a string representation of the Polynomial using exponential notation for the coefficients and with the specified number of <i>digits</i> . Notice the <i>digits</i> is optional
toFixed(digits)	Return a string representation of the Polynomial where the coefficients contains a specified number of <i>digits</i> after the decimal place. Notice the <i>digits</i> is optional
toPrecision(digits)	Return a string representation of the Polynomial where the coefficients contains either exponential or fixed point notation depending on the size of the number and the number of significant <i>digits</i> specified.  Notice the digits is optional
toString()	Return a string representation of the Polynomial object
valueof()	The primitive value number of this Polynomial object.

# Methods

Methods	Descriptions
add(a,b)	Add two Polynomial object together

compositedeflate(z)	Deflate a Polynomial with the root z using composite
	deflation
deflate(z)	Deflate a Polynomial with the root z
derivative()	Return the derivative polynomial
div(a,b)	Return the division of two polynomial
mul(a,b)	Return the multiplication of two polynomial
pow(p,n)	Return the power of raising the Polynomial to n
rem(a,b)	Return the remainder polynomial after dividing the
	polynomial a/b
sub(a,b)	Returned the subtracted polynomial a-b
value(z)	Returned the value of the Polynomial at point z
zeros()	Find all zeros of a Polynomial

# Constants

zero return a new Polynomial() object one return a new Polynomial(1) object

# Miscellaneous

parsePolynomialt() Parse a Polynomial string and return a Polynomial object

# Polynomial.array()

Return the Polynomial object coefficients as an Array of coefficients

# **Synopsis**

Polynomial object.array()

### Returns

The coefficients of the Polynomial object is returned in the return array

# **Example**

```
var p = new Polynomial(1,2,3); // x^2+2x+3
var coeff;
var coeff=p.array() // coeff=[1,2,3]
```

#### See Also

Polynomial.join()

# Polynomial.add()

Add two Polynomials

# **Synopsis**

Polynomial.add(a,b)

# Arguments

*a,b* The Polynomials to be added.

### Returns

The result of the Polynomial addition a+b.

```
var x=new Polynomial(1,2,3); // x^2+2x+3
var y=new Polynomial(5,6); // 5x+6
var z=new Polynomial(Complex(1+i),Complex(2-2i),3); // (1+i)x^2+(2-2i)x+3
```

```
var p=Polynomial.add(x,y) // result x^2+7x+9
var p2=Polyomial.add(z,x) // result (1+i)x^2+(4-2i)x+6
```

#### See Also

Polynomial.div(), Polynomial.mul(), Polynomial.sub(), Polynomial.rem()

# Polynomial.compositedeflate()

Deflate a root of the Polynomial using composite deflation

## **Synopsis**

Polynomial object.compositedeflate(z)

## **Arguments**

z The root by which the Polynomial is composite deflated.

#### Returns

No returns

### Result

The *Polynomial Object* has been deflated with the root z. Notice the deflation is done using composite deflation meaning dividing the root z into Polynomial using both a forward and backward deflation and then determine for which power  $x^y$  to begin using the backward deflated coefficients to minimize the division error and using forward deflated coefficients for power higher than  $x^y$ . For the coefficients at  $x^y$  the coefficient is calculated as the average between the forward and backward deflated coefficient for  $x^y$ .

### **Example**

```
var p = new Polynomial(1,-6,11,-6);// 1x^3-6x^2+11x-6
var x = 2; // one root is 2
p.compositedeflate(x); // result x^2-4x+3
```

## See Also

Polynomial.deflate()

# Polynomial.degree()

Return the degree of the Polynomial object

# **Synopsis**

Polynomial object.degree()

### Returns

The degree of the Polynomial object.

# **Example**

```
var p = new Polynomial( 1,-6,11,-6 );// 1x^3-6x^2+11x-6 var n;
```

n=p. degree(); // n = 3

### See Also

Polynomial.getcoeff(), Polynomial.setcoeff()

# Polynomial.deflate()

Deflate a root of the Polynomial using forward deflation

# **Synopsis**

Polynomial object.deflate(z)

# Arguments

z The root by which the Polynomial is forward deflated.

#### Returns

No returns

### Result

The *Polynomial Object* has been deflated with the root z. Notice the deflation is done using forward deflation meaning dividing the root z into Polynomial starting with the coefficients with the highest power. E.g.  $x^n$ . If the roots are deflated, using increasing magnitude of the root the forward deflation method is numerical stable. If in doubt a root

cant been guarantee to be deflated in increasing order of magnitude then use the composite deflation method.

# Example

```
var p = new Polynomial(1,-6,11,-6);// 1x^3-6x^2+11x-6 var x = 2;  // one root is 2 p. deflate(z);  // result x^2-4x+3
```

### See Also

Polynomial.compositedeflate()

# Polynomial.derivative()

Calculate the derivative coefficients of the Polynomial object

# **Synopsis**

Polynomial object.derivative()

# **Arguments**

none

### Returns

Return a new Polynomial, which is the derivative of the Polynomial object.

# **Example**

```
var p = new Polynomial(1,-6,11,-6);// 1x^3-6x^2+11x-6
var dp=p. derivative(); // dp is 3x^2-12x+11 same as Polynomial(3,-12,11);
```

# See Also

# Polynomial.div()

Divide two Polynomial numbers

# **Synopsis**

Polynomial.div(a,b)

# **Arguments**

a,b The Polynomials to be divided.

## Returns

The result of the Polynomial division a/b.

# Example

```
var x = new Polynomial(1,-6,11,-6); // 1x3-6x2+11x-6
var y = new Polynomial(1,-2); // x-2
Polynomial.div(x,y) // result x^2-4x+3
```

### See Also

Polynomial.add(), Polynomial.mul(), Polynomial.sub(), Polynomial.rem()

# Polynomial.getcoeff()

Get one Polynomial coefficients

# **Synopsis**

*Polynomial object*.getcoeff( $x^{th}$ )

# **Arguments**

 $x^{th}$  The coefficient to the  $x^{th}$  degree.

### Returns

Return the coefficient associated with the  $\mathbf{x}^{\text{th}}$  degree of the Polynomial object.

# Example

```
var p = new Polynomial( 1,-6,11,-6 );// 1x^3-6x^2+11x-6 var coeff;
coeff=p. getcoeff(2); // coeff=-6
```

# See Also

Polynomial.setcoeff()(), Polynomial.degree()

# Polynomial.isComplex()

Determine if the Polynomial object contains any complex numbers

# **Synopsis**

Polynomial object.isComplex()

#### Returns

Return true if the Polynomial object contains any coefficients that is a complex number otherwise false.

# Example

```
var p = new Polynomial(1,2,3); // x^2+2x+3
var complexnumber;
var complexnumber=p.isComplex() // return false;
```

# See Also

Polynomial.isReal()

# Polynomial.isReal()

Determine if the Polynomial object contains all real numbers

# **Synopsis**

Polynomial object.isReal()

### Returns

Return true if the Polynomial object coefficients contains all real number otherwise false.

```
var p = new Polynomial(1,2,3); // x^2+2x+3
var onlyreal;
var onlyreal=p.isReal() // return true;
```

# See Also

Polynomial.isComplex()

# Polynomial.join ()

Return the Polynomial object coefficients as a join String

# **Synopsis**

Polynomial object.join(separator)

# **Arguments**

separator Separator character. If omitted the default separator is ","

### Returns

The coefficients of the Polynomial object is returned as a joined string using the separator between coefficients.

# Example

```
var p = new Polynomial(1,2,3); // x^2+2x+3
var str;
var str=p.join() // str="1,2,3"
```

# See Also

Polynomial.array()

# Polynomial.monic()

Bring the Polynomial object into a monic form

# **Synopsis**

Polynomial object.monic()

## Returns

A monic Polynomial object where the leading coefficient to  $a_nx^n$  is scaled to 1. Taking  $a_n$  and divide it up in the other coefficients  $a_{n-1},...a_1,a_0$ . The same effect can also be archived by using the property *Polynomial*.scale( $1/a_n$ ).

# **Example**

```
var p = new Polynomial(2,3,4); // 2x^2+3x+4
p.monic () // p is now x^2+1.5x+2
```

### See Also

Polynominal.scale()

# Polynomial.mul()

Multiply two Polynomials

# **Synopsis**

Polynomial.mul(a,b)

# Arguments

a,b The Polynomials to be multiplied.

### Returns

The result of the Polynomial multiplication a\*b.

# Example

```
var x = new Polynomial(1,-1); // x-1

var y = new Polynomial(1,-4,3); // x^2-4x+3

z=Polynomial.mul(x,y) // x^3-6x^2+11x-6
```

## See Also

Polynomial.add(), Polynomial.div(), Polynomial.sub(), Polynomial.rem()

# Polynomial.normalize ()

Normalize the Polynomial object

# **Synopsis**

Polynomial object.normalize()

### Returns

A normalized Polynomial object. This mean removing leading or trailing zeros. Any undefined coefficients is converted to 0.

# Example

# See Also

# Polynomial.one

Return the constant one as a Polynomial object

# **Synopsis**

Polynomial.one

### Returns

The Polynomial constant one object. Same as New Polynomial(1).

# **Example**

```
var x = Polynomial.one; // x=1 x is a Polynomial object
```

## See Also

Polynomial.zero

# Polynomial.parsePolynomial()

Parse and convert a Polynomial string into a Polynomial object

# **Synopsis**

parsePolynomial(string)

# Arguments

string String to be parsed into a Polynomial object

#### Returns

parsePolynomial() parses and return a new Polynomial object contained in s. parsePolynomial() return an empty Polynomial object if parsing fails. A Polynomial object followed the standard syntax:

```
a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0
```

where the coefficients  $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$  can be either a regular integer, floating point number as in JavaScript or a complex JavaScript number following the complex syntax as outline in the Complex number JavaScript library packages. Notice the format for  $x^n$  need to be expressed in a string as  $x^n$ .

Furthermore you can use Polynomial arithmetic (\*,+,-,/), grouping with () and power with the  $^{\circ}$  operator, see example below. The returned Polynomial object is guarantee to not have any undefined coefficients. E.g x $^{\circ}$ 5-1 is the same as new Polynomial(1,0,0,0,0,-1);

.

# Example

```
 \begin{array}{ll} \text{var p = parsePolynomial("$x^2$+$3$x$+$6");} & \text{// Same as Polynomial(1,3,6) or $x^2$+$3$x$+$6} \\ p = parsePolynomial("$-2x^2$+$3$x$-$5"); & \text{// Same as Polynomial(-2,3,-5);} \\ p = parsePolynomial("$(1x^2$+$2x$+$3)^2"); & \text{// Same as Polynomial(1,4,10,12,9);} \\ p = parsePolynomial("$(x$-$1)(x$-$2)(x$-$3)"); & \text{// Same as Polynomial(1,-6,11,-6);} \\ \end{array}
```

#### Notice

The string can also contains Polynomial expression with the operator  $+,-,*,/,\%,^{\wedge}$  and arithmetic grouping with (). E.g.

"(x-1)(x-2)\*(x^3-1)^5" or "(x-1)^15" are all valid strings that can be converted by parsePolynomial() into a Polynomial object. Complex number can also be handle e.g. "(3-i4)x^3+(-2)x^2+(-i4)x-(3)" notice you would need to group them using () as coefficient to  $x^n$ 

Also notice you do not need the \* operator in from of a () as it also interpret implicit multiplication correctly.

### See Also

new Polynomial(), Polynomial()

## Polynomial.pow()

Compute p<sup>y</sup> where p is a Polynomial

#### **Synopsis**

Polynomial.pow(p,n)

# Arguments

p A Polynomial object to be raised to a power

*n* The power that *p* is raised to. n need to be a positive integer

#### Returns

x to the power of y or  $x^y$ 

# Example

```
var p = new Polynomial(1,2,3); // x^3+2x+3
var n = 2;
```

Polynomial.pow(p,2);  $// x^4+4x^3+10x^2+12x+9$ 

### See Also

Polynomial.mul()

# Polynomial.rem()

Divide two Polynomial numbers and return the remainder Polynomial

# **Synopsis**

Polynomial.rem(a,b)

# **Arguments**

a,b The Polynomials to be divided.

### Returns

The remainder Polynomial as a result of the division a/b.

```
var x = new Polynomial(1,-6,11,-6); // 1x3-6x2+11x-6
var y = new Polynomial(1,-2); // x-2
```

var z=Polynomial.rem(x,y) // result 0 because x is dividable by y with a zero remainder

### See Also

Polynomial.add(), Polynomial.mul(), Polynomial.sub(), Polynomial.div()

## Polynomial.scale ()

Scale the Polynomial object

## **Synopsis**

*Polynomial object*.scale(scale)

## Arguments

scale

Optional parameter with the scale factor. If omitted an auto scaling is performed.

#### Returns

A scaled Polynomial object where the all coefficients of the Polynomial are multiplied by scale parameter. If scale parameter is omitted it is auto scaled based on very large or very small coefficients. Computes a scale factor to multiply the coefficients of the polynomial. The scaling is done to avoid overflow and to avoid undetected underflow interfering with the convergence criterion.

If auto scaled, the scale factor is a power of the base (2) to avoid loss of precision.

# Example

```
var p = new Polynomial(2,3,4); // 2x^2+3x+4
p.scale (0.5) // p is now x^2+1.5x+2
```

## See Also

### Polynomial.setcoeff()

Set one Polynomial coefficient

### **Synopsis**

*Polynomial object*.setcoeff(*x*<sup>t</sup>,*newcoeff*<sup>h</sup>)

## Arguments

 $x^{th}$  The coefficient to the  $x^{th}$  degree. newcoeff The replacement value for the  $x^{th}$  coefficient

### Returns

Return the new coefficient associated with the x<sup>th</sup> degree of the Polynomial object.

# Example

```
var p = new Polynomial( 1,-6,11,-6 );// 1x^3-6x^2+11x-6 var coeff; coeff=p. setcoeff(2,8); // coeff=8 and Polynomial is 1x^3+8x^2+11x-6
```

### See Also

Polynomial.getcoeff()(), Polynomial.degree()

## Polynomial.shift()

Do a Polynomial Taylor shift of the Polynomial resulting in new coefficients

# **Synopsis**

*Polynomial object*.shift(*n*)

### Returns

Do a Polynomial Taylor shift of offset n. The new Polynomial has the same roots as the original Polynomial shift n to the left. E.g. if a root was 2 prior to shift then *Polynomial object*.shift(1) result in the new polynomial has a root of 1. The original Polynomial root can be recreated by adding the shift n, to all the root and these roots are the roots of the original Polynomial.

# Example

```
var p = new Polynomial( 1,0,0,0,-1 ); // 1x^{5-1}
p. shift(1); // Polynomial is now 1x^5+5x^4+10x^3+10x^2+5x
```

### See Also

Polynomial.scale()

# Polynomial.simplify()

Simplify the Polynomial coefficients

# **Synopsis**

Polynomial object.simplify()

### Returns

Simplify Polynomial where all complex number coefficients with an imaginary part of 0 has been converted into a real number.

# Example

```
var p = new Polynomial( 1,New Complex(-6,0),11,-6 ); // 1x^3(-6+i0)x^2+11x-6
p. simplify(); // Polynomial is now 1x^3-6x^2+11x-6
```

### See Also

Polynomial.getcoeff()(), Polynomial.degree()

# Polynomial.sub()

Subtract two Polynomials

# **Synopsis**

Polynomial.sub(a,b)

# Arguments

*a,b* The Polynomials to be subtracted.

## Returns

The result of the Polynomial subtraction.

# Example

### See Also

Polynomial.add(), Polynomial.div(), Polynomial.mul(), Polynomial.rem()

## Polynomial.toExponential()

Format a Polynomial using exponential notation for the coefficients

# **Synopsis**

Polynomial.toExponential(digits)

## **Arguments**

Digits

The number of digits that will appear after the decimal point. This may be a value between 0 and up. If this argument is omitted, as many digits as necessary will be used.

### Returns

A string representations of the Polynomial, where all coefficients are in exponential notation, with one digit before the decimal place and *digits* digits after the decimal place. The fractional part of the Polynomial coefficients number is rounded, or padded with zeros, as necessary, so that is has the specified length.

## Example

```
var p=new Polynomal(10.5567,1.66,-200) //10.5567x<sup>2</sup>+1.66x-200
p.toExponential(1); // 1.1e+2x<sup>2</sup>+1.7e+0-2e+2
p.toExponential(3); // 1.056e+1x<sup>2</sup>+1.66x-2e+2
p.toExponential(); // 1.05567e+1x<sup>2</sup>+1.66x-2e+2
```

#### See Also

Polynomial.toFixed(), Polynomial.toPrecision(), Polynomial.toString()

# Polynomial.toFixed()

Format a Polynomial using fixed-point notation for the coefficients

## **Synopsis**

*Pylonomial*.toFixed(digits)

# Arguments

**Digits** 

The number of digits that will appear after the decimal point for the Polynomial coefficients. This may be a value between 0 and 20, inclusive. If this argument is omitted, it is treated as zero.

#### Returns

A string representations of the Polynomial, that does not used exponential notation and has exactly *digits* digits after the decimal point. The *Polynomial coefficients* is rounded as necessary, and the fraction part is padded with zeros if necessary so that it has the specified length. If the *Polynomial coefficients* is greater than 1e+21, this method simple calls *Polynomial*.toString() and return a string in exponential notation.

# Example

### See Also

Pylonomial.toExponential(), Polynomial.toPrecision(), Pylonomial.toString()

# Polynomial.toPrecision()

Format the significant digits of a Polynomial coefficients

# **Synopsis**

Polynomial.toPrecision(digits)

# Arguments

Digits

The number of significant digits to appear as the coefficients in the returned string. This may be a value between 1 and 21, inclusive. If this argument is omitted, the toString() method is used instead.

#### Returns

A string representations of the *Polynomial*, that contains *precisions* significant digits in the cofficients. If *precision* is large enough to include all the digits of the integer part of number, the returned string uses fixed-point notation. Otherwise, exponential notation is used with one digit before the decimal place and *precision* – 1 digits after the decimal place. The number is rounded or padded with zeros as necessary.

# Example

```
var p=new Polynomal(10.5567,1.66,-200) //10.5567x<sup>2</sup>+1.66x-200
p.toFixed(1); // 1.1e+1x<sup>2</sup>+1.7-2e+2
p.toFixed(3); // 1.557e+1x<sup>2</sup>+1.66x-200
```

### See Also

Polynomial.toExponential(), Polynomial.toFixed(), Polynomial.toString()

# **Polynomial.toString()**

Format the significant digits of a Polynomial coefficients

# **Synopsis**

*Polynomial*.toString(radix)

## Arguments

Radix

If omitted the base 10 will be used to convert the Polynomial coefficients to a string. Otherwise the radix will be used (2..36).

## Returns

A string representations of the *Polynomial*, in the indicated radix, returned as a normalized number.

```
var p=new Polynomal(10.5567,1.66,-200) //10.5567x<sup>2</sup>+1.66x-200
p.toString(); // //10.5567x<sup>2</sup>+1.66x-200
```

### See Also

Polynomial.toExponential(), Polynomial.toFixed(), Polynomial.toPrecision()

# Polynomial.valueof()

Return the primitive value of the coefficients as an array

# **Synopsis**

Polynomial object.valueof()

## Returns

The primitive value of the *Polynomial* is returned in normalized form.

# Example

```
var p = new Polynomial(1,2,3); //x^2+2x+3
p.valueOf() // return 1
```

### See Also

# Polynomial.value()

Return the value of the Polynomial at point x.

# **Synopsis**

*Polynomial object*.value(z)

# **Arguments**

z is the point at which to calculate the value of the Polynomial object. z can be an integer, floating point or complex number.

### Returns

Return the value of the Polynomial at point z.

```
var p = new Polynomial(1,-5,6); //x^2-5x+6
p.value(1) // return 2
```

### See Also

# Polynomial.zero

Return an Empty Polynomial object

# **Synopsis**

Polynomial.zero

#### Returns

The Polynomial object where the coefficients is undefined

# Example

var p = Polynomial.zero;

# See Also

Polynomial.one

# Polynomial.zeros()

Find all the roots of the Polynomial.

# **Synopsis**

Polynomial object.zeros(method, verbose, composite)

# **Arguments (Notice all arguments is optional)**

method Select the method to use for finding the zeros. Currently the following

methods are supported:

"Newton", "Ostrowski", "Halley", Householder".

If method is undefined it will default to Newton's method.

verbose If verbose is true verbose information of how the root finding progress is

generated and return as Array[0] in the return Array. If verbose is

undefined it defaults to false;

composite If composite is true the deflation is done using the composite deflation

method. Since we find the root in increasing magnitude the default

forward deflation method is just as an accurate as using the composite deflation method. If composite is undefined it defaults to false;

### Returns

Return all the zeros of the Polynomial object as an Array. For a polynomial with degree n the roots in the Array is from 1..n for a total of n roots. Array [0] contains the verbose information generated as a textual string.

# Example

```
var p = new Polynomial(1,-5,6); //x^2-5x+6
var x=p.zeros("Newton",false,false); // return
// x[2]=1.9999999999999997,
// x[1]=3.00000000000000042
// x[0]=""; verbose is false;
```

# See Also